Incorporation of Critical Thinking Skills Development: A Case of Mathematics Curriculum for Grades I-XII

Muhammad Jamil[1], Tahira Batool Bokhari[2] and Javed Iqbal[3]

https://doi.org/10.62345/jads.2024.13.1.32

Abstract

This research explores the analysis of the National Curriculum for Mathematics (2006) in Pakistan to promote critical thinking among students from grades I to XII. The qualitative content analysis was used to systematically scrutinize the curriculum document to determine significant aspects that contribute to developing critical thinking skills. The analysis focused on developing conceptual understanding and skill development while addressing real-life problem-solving and logical reasoning. It is based on a spiral approach where concepts are revisited and deepened across grades, promoting interactive, student-centered teaching methods. Finally, it highly underscores the use of technology, especially at higher levels, that can enhance mathematical exploration. The curriculum also demonstrates its commitment to reasoning and logical thinking by dedicating a separate standard and incorporating activities encouraging pattern recognition, argumentation, and justification. However, how successful this can be in classrooms will depend primarily on how well it is executed. This study provides valuable insights into the potential of the mathematics curriculum in Pakistan to develop student's critical thinking abilities and offers recommendations for further improvement. The findings imply that designing curricular materials is essential in realizing 21st-century skills; remarkably, the National Curriculum for Mathematics (2006), Pakistan, is well-grounded in assisting students' development regarding critical thinking.

Keywords: Critical Thinking, Mathematics Curriculum, Qualitative Content Analysis, Curriculum Design.

Introduction

In education, fostering critical thinking skill development is essential as it equips learners with knowledge assessment techniques (Paul & Elder, 2019). This is even more important in Mathematics, as problem-solving and logical reasoning form the basis of this field (NCTM 2000). One area that has recently sparked interest in educational discourse is integrating math curricula across all classes between one and twelve, focusing on nurturing students' critical thinking abilities. The current study explores how Pakistan's National Curriculum for Mathematics (2006) can help students at different grade levels develop critical thinking skills.

Critical thinking is a vital skill that helps individuals solve complex problems, make sound decisions, and engage meaningfully with their environment (Ennis, 2018). This study focuses on critical thinking in students within the context of Mathematics education as it will help them

[1]*Lecturer, Department of Education, GC Women University Sialkot, Pakistan. Email: m.jamil@gcwus.edu.pk*
[2]*Associate Professor, Government College for Women, Jhang.*
[3]*Associate Professor, Department of Education, Ghazi University, Dera Ghazi Khan. Email: jdiqbal@gudgk.edu.pk*

Compliance with ethical standards: There are no conflicts of interest (financial or non-financial). This study did not receive any funding.

understand mathematical concepts fully, apply them in different situations, and thus build a solid foundation for further learning (Tall, 2013). Mathematical curriculum may be enriched by incorporating components related to critical thinking into it. Teachers can improve their students' cognitive capacities, analytical abilities, and general academic performance by including critical thinking components in the Mathematics curriculum (Facione et al., 2020).

The National Curriculum for Mathematics (2006) outlines the topics to be learned, objectives to be met, and how instruction should be carried out from grade I up to grade XII. It is a Mathematics curriculum for grades I through XII entitled "The National Curriculum for Mathematical Education (2006)", which specifies what should be taught in Pakistan's classrooms and how teachers should teach it. The curriculum focuses heavily on helping students learn to think critically, reason logically, and problem-solve, apart from gaining math knowledge (Ministry of Education, 2006). A qualitative content analysis was used to examine the curriculum document and identify key features supporting critical thinking development in teaching Mathematics.

One notable thing about the National Curriculum for Mathematics (2006) is that it emphasizes conceptual knowledge-building among learners. The curriculum requires numerous visitations and deeper understanding across levels to establish an interactive student-centered approach. Finally, technology integration, especially in advanced stages such as at the higher school level, can also promote a more thorough exploration of mathematical aspects. There are other explanations for logical reasoning since another standard is dedicated to reasoning and logical thoughts incorporated with pattern recognition, argumentation, and justification. However, the curriculum can only develop critical thinking skills if it is successfully implemented in classrooms. The curriculum emphasizes real-world problem-solving scenarios and logical reasoning exercises to establish a solid understanding of mathematical ideas while enhancing students' critical thinking skills (Ministry of Education, 2006). Using the spiral method in the curriculum ensures repeated reviews and reinforcement of core ideas at different grades, which contributes to the growth of students' critical thinking ability (Tirol, 2022).

The National Curriculum for Mathematics (2006) is based on an interactive teaching methodology, which aims to foster active student engagement through activities that promote critical thinking. On top of this line, the curriculum seeks to engage students actively in the learning process by supporting them with critical thinking abilities through active participation by learners in collaborative learning groups and hands-on inquiry experiences (Makonye, 2019). Also, using technological tools and resources can enhance one's math experiences and strengthen one's critical reasoning abilities or problem-solving capabilities, according to Safitri and Aziz (2022). This is especially true for higher-grade students. In addition, The National Curriculum for Mathematics (2006) introduces technological instruments that can be used to support young people's discovery of Mathematics and foster their logical thinking skills. By incorporating digital developments into classroom practices, teachers could create dynamic learning environments that facilitate interactive problem-solving tasks, data-analyzing methods, and innovative techniques (Kwangmuang et al., 2021). Furthermore, some parts of the syllabus explicitly guide students' thinking so that they become able to pattern recognition. This encourages them to make an argument, justify their decisions, and develop many cognitive processes that help them think critically (Ministry of Education, 2006).

Scholars and educators worldwide have focused on developing critical thinking skills in Mathematics education. Many studies have tried to discover why it is crucial to develop critical thinking skills among students and techniques for embedding them in Mathematics syllabuses. Aizikovitsh-Udi and Cheng (2015) argue that critical thinking is essential beyond school walls in

the real world. They also state that learners must apply what they learn and be able to use it in their lives. Furthermore, Math lessons should include teaching strategies for high-order thinking skills within the math curriculum (Basri & As' ari, 2019).

New studies about technology's role in fostering critical thinking in Mathematics education are emerging. Kiru et al. (2018) have researched how technology enhances learning environments and promotes secondary school students' critical reasoning capacity when solving math problems. They found that students solve math problems more logically with interactive digital tools. Similarly, Mao et al. (2022) investigated the impact of game-based learning on students' critical thinking skills. A meta-analysis study found a positive effect of game-based learning on students' critical thinking skills. The importance of teacher professional development in promoting critical thinking skills in Mathematics education is highlighted by (Firdaus & Darari, 2024). The purpose of the study was to explore the effectiveness of case-based interactive videos with online discussion forums for the improvement of CT skills of prospective teachers at the university level. The findings showed a significant increase in scores in CT skills in a post-test. On the other hand, the qualitative results highlighted the significance of collaboration and engagement in the learning process.

Several studies have explicitly focused on integrating critical thinking skills within curricula in the Pakistani context. A study conducted by Jamil (2024) regarding the analysis of the Chemistry curriculum for grades IX-X, the analysis of the Biology curriculum fostering CT skills for the twenty-first century (Jamil, Bookhari, et al., 2024), the Physics curriculum concerning the incorporation of CT skills development (Jamil & Muhammad, 2024), Single National Curriculum (SNC), 2020 for Social Studies Grades IV-V for the development of CT (Jamil et al., 2024), teaching of CT in Pakistan studies textbook for grade 9 (Naseer et al., 2022), overall analysis of physics, chemistry, and biology curriculum for the development of CT skills among secondary school students (Jamil et al., 2021). All these studies analyze the different curriculum documents regarding CT skills development as incorporated in the documents with a strong emphasis on CT, its importance, suggested pedagogical practices, and assessment systems for the development of CT skills development.

Objective of the Study

To explore the critical thinking skills development incorporated in the Mathematics curriculum (2006) Grades I-XII.

Research Methodology

This study employed a qualitative content analysis approach to investigate the potential of a widely used Mathematics curriculum in fostering critical thinking skills among Grades I-XII students. Qualitative content analysis is a research method used to identify patterns, themes, and other ideas through text data analysis (Kyngäs, 2020). This method best analyzes educational material like textbooks to see if their content affects learning outcomes (Mayring, 2014). The Mathematics curriculum for grades I-XII was reviewed using a purposive sampling technique. This non-probability method finds participants based on specific commonalities or characteristics between them (Etikan & Bala, 2017). The entire curriculum document was analyzed word-to-word. The document was obtained digitally, which made it easier to collect and organize data. NVivo 12 software was used for the qualitative content analysis of this study (Bazeley & Jackson, 2019).

Findings of the Study

Based on a thorough analysis of the National Curriculum for Mathematics document from grades I-XII in Pakistan, here is a detailed review of different critical aspects with relevant examples and page number references:

1. Emphasis on Conceptual Understanding and Skill Development

The curriculum strongly emphasizes building a conceptual understanding of mathematical ideas and developing essential skills. The curriculum is designed to help students ″build the solid conceptual foundation in Mathematics that will enable them to apply their knowledge skillfully" (p. 2). The "curriculum emphasizes the geometrical concepts that enable the students to think logically, reason systematically, and conjecture astutely″ (p. 2). The standards include "conceptual understanding – comprehension of mathematical concepts, operations and relations" and "procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently and appropriately" as critical components of mathematical proficiency″ (p. 132).

2. Focus on Real-life Problem Solving

The curriculum stresses the importance of connecting Mathematics to real-life situations and developing problem-solving abilities in students. For instance, the curriculum notes, "an information- and technology-based society requires individuals, who are able to think critically about complex issues, analyze and adapt to new situations, solve problems of various kinds and communicate their thinking effectively" (p. 1). The teaching strategies section states, "problem solving is a widespread opinion that should be the central focus of the curriculum for Mathematics" and "working on this problem offers a good practice in addition, multiplication and division skills. But the important goal of this problem is to help students think systematically about possibilities and record thinking" (p. 133). Across the grades, many word problems and problem-solving exercises are included in the content. For example, in Grade III, students are expected to "solve real life problems related to time and work using proportion" (p. 53*).*

3. Integration of Technology

Recognizing the importance of technology in Mathematics education, the curriculum includes using technology at appropriate places. For example, the curriculum "recognizes the benefits that current technologies can bring to the learning and doing Mathematics. It, therefore, integrates the use of appropriate technologies to enhance learning in an ever increasingly information-rich world" (p. 2). Specific software like MAPLE is introduced in higher grades to "integrate technology to aid the process of mathematical exploration" (p. 4-6). The use of technology like calculators, graphing tools, dynamic geometry software, etc., is suggested at relevant places.

4. Spiral Approach to Concepts

The curriculum is structured in a spiral manner, with concepts introduced in earlier grades and then revisited in higher grades with increasing depth and complexity. For example, the concept of whole numbers is introduced in grade I (p. 8), extended to 6-digits in grade IV (p. 24), and further to 100 million in grade V (p. 32). Similarly, the topic of fractions is introduced in grade I (p. 11), extended to all four operations by grade IV (p. 25-26) and further developed in grade V with concepts like a fraction to decimal conversion (p. 33).

5. Emphasis on Reasoning and Logical Thinking

The curriculum substantially emphasizes developing students' students' reasoning and logical thinking abilities. One of the five primary standards is dedicated to "reasoning and logical thinking," with the objective that students should be able to "use patterns, known facts, properties, and relationships to analyze mathematical situations" and "examine real-life situations by identifying, mathematically valid arguments and drawing conclusion to enhance their mathematical thinking" (p. 7). Across grades, reasoning skills are developed through exercises involving pattern recognition, analyzing mathematical situations, evaluating arguments, justifying solutions, etc. For instance, in grade VIII, students are expected to "find different ways of approaching a problem to develop logical thinking and explain their reasoning" (p. 7).

6. Interactive Teaching Approach

The curriculum advocates an interactive, student-centered teaching approach that moves away from simple lecture-based methods. It specifies, "The teachers' role shifts from dispensing information to planning investigative tasks, managing a cooperative learning environment and supporting students' creativity in developing a rational understanding of the concepts" (p. 132). It is also described as "Students particularly of the primary level can learn by presentation and explanation by the teacher, consolidation and practice, games, practical work, problems, and puzzles, and investigating Mathematics" (p. 133). Problem-solving is also discussed.

Discussion

To nurture critical thinking skills among students from grades I to XII, this study's findings highlight the potential of the National Curriculum for Mathematics (2006) in Pakistan. Also, the spiraling progression of concepts, reasoning and logical thinking, real-life problem solving, interactive teaching methods, technology integration, conceptual understanding focus, and skill development, just like many other literatures, aligns with this emphasis of curriculum. Like Tall (2013), the curriculum mainly stresses building a solid mathematical foundation that helps understand concepts. Firstly, the curriculum is geared towards helping students understand Mathematics concepts, including operations and relations, necessary to develop their CT skills. This further confirms Rittle (2017), who found that conceptual understanding is vital when solving problems and transferring learning.

According to Enu and Ngcobo (2022), this curriculum's use of real-life problem-solving scenarios is observed in teachers' knowledge and practices on feedback, which were based on corrective measures given a thorough analysis of student weakness. Relating mathematical ideas to real-life situations can encourage the application of knowledge by pupils analyzing complex problems and devising alternative solutions (Schoenfeld, 2022). The need for authentic problem-solving experiences in Mathematics education has also been highlighted by Schoenfeld (2022).

With the growing recognition that technology might improve critical thinking abilities, technology incorporation into the National Curriculum for Mathematics (2006) conforms to such recognition. Since it demonstrates positivity regarding mathematically reasoned ideas and problem-solving capabilities amongst learners, research conducted by Bray and Tangney (2017) reveals how its usage aligns with MAPLE software. Drijvers (2020) also asserts that digital tools can promote conceptual understanding through exploration and visualization. The emphasis placed on reasoning and logical thinking within the framework finds support from Ciccolini and Stylianides (2020); they stated that it was important for teachers to develop pupils' reasoning skills. Therefore,

the curriculum aims to improve their critical thinking through participation in activities like looking for patterns, analyzing mathematical situations, and justifying answers.

The curriculum's advocacy for interactive and student-centered teaching strategy is congruent with the principles of constructivism and the findings of several studies that underline the benefits of active learning in Mathematics education (Asunka et al., 2018). By allowing scientific investigations and group projects and encouraging creativity in students' work, a better learning environment for nurturing critical thinking skills is aimed. However, some obstacles can occur while implementing the curriculum. According to Amirali and Halai (2010), teachers' training, adequate resources, and resistance to change have hindered the implementation of curriculum reforms in Pakistan. These obstacles need attention to achieve the goals set by the curriculum.

Besides, this study has shown that continuously evaluating and taking the curriculum is necessary. According to Confrey et al. (2018), curriculum development should be an iterative process incorporating feedback from educators, students, or other stakeholders. Thus, constant assessment and adaptation of a syllabus according to new research and shifting educational demands can keep its relevance and efficiency in enhancing critical thinking skills.

Conclusion

The current study analyzed the Mathematics curriculum in terms of fostering CT skills among students. Qualitative content analysis was used to analyze the curriculum to focus on conceptual understanding of skill development, real-life problem solving, technological integration, spiral approach for concepts, and logical and reasoning thinking with an interactive teaching approach. The study results emphasize the sound foundation of mathematical concepts and improving critical thinking skills. Through conceptual understanding, the curriculum enables the students to use mathematical knowledge effectively. Furthermore, in real-life situations, Mathematics is discussed in the curriculum to solve problems among students. Throughout all grades, problem-solving exercises are included to encourage the students to think critically regarding complex issues. To improve their mathematical thinking capacities by involving them in activities like pattern recognition evaluation of arguments used in analyzing mathematical situations or justifying solutions made by them during a particular lesson, etc., the authors claim that such activities would help learners recognize numbers and think mathematically better than before. Also worth mentioning is that an interactive teaching method where students' interests are taken into consideration was proposed by The National Curriculum for Mathematics (2006). A more interactive and efficient learning environment is created with investigative tasks replacing teacher-centered instruction and cooperative learning supportive of students' creativity. The recommended teaching strategies, such as investigations, explorations, discussions, projects, and manipulatives, encourage students to actively participate in their learning processes and gain a deeper understanding of mathematical concepts.

Recommendations

The following are the study's recommendations, keeping the findings and conclusions in view.

- Comprehensive teacher training programs should be conducted focusing on pedagogical practices for the development of CT skills.
- Educational resources, including textbooks, technological tools, and teaching material, should be provided to implement the curriculum across all grades.
- Technology integration in Mathematics classrooms should be encouraged to foster CT skills among students.

- Connecting mathematical concepts with real life should be encouraged to make learning more relevant.
- Assessment strategies should be developed and implemented in a way that is aligned with the curriculum.

References

- Aizikovitsh-Udi, E., & Cheng, D. (2015). Developing critical thinking skills from dispositions to abilities: mathematics education from early childhood to high school. *Creative Education*, *6*(4), 455-462.
- Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A survey of secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research*, *32*(2), 45.
- Asunka, S., Freeman, E., & Arthur, L. S. (2018). *Implementing constructivist pedagogy in a flipped mode in a postgraduate course.* ICERI2018 Proceedings,
- Basri, H., & As' ari, A. R. (2019). Investigating Critical Thinking Skill of Junior High School in Solving Mathematical Problem. *International Journal of Instruction*, *12*(3), 745-758.
- Bazeley, P., & Jackson, K. (2019). Qualitative data analysis with NVivo. *Qualitative data analysis with NVivo*, 1-376, Sage.
- Bray, A., & Tangney, B. (2017). Technology usage in mathematics education research–A systematic review of recent trends. *Computers & Education*, *114*, 255-273.
- Ciccolini, G., & Stylianides, G. J. (2020). Teaching to listen, question, and ponder: An investigation into promoting an enquiring atmosphere in a mathematics classroom. *Journal of Educational Research in Mathematics, 3*(SP1), 29-68.
- Confrey, J., Maloney, A. P., Belcher, M., McGowan, W., Hennessey, M., & Shah, M. (2018). The concept of an agile curriculum as applied to a middle school mathematics digital learning system (DLS). *International Journal of Educational Research*, *92*, 158-172.
- Drijvers, P. (2020). Digital tools in Dutch mathematics education: A dialectic relationship. *National Reflections on the Netherlands Didactics of Mathematics: Teaching and Learning in the Context of Realistic Mathematics Education*, 177-195.
- Ennis, R. H. (2018). Critical thinking across the curriculum: A vision. *Topoi*, *37*, 165-184.
- Enu, J., & Ngcobo, A. Z. (2022). Connecting knowledge and practice: Mathematics teacher educators' knowledge and use of formative feedback in Ghana. *Journal of Education (University of KwaZulu-Natal)*(87), 71-92.
- Etikan, I., & Bala, K. (2017). Combination of probability random sampling method with non probability random sampling method (sampling versus sampling methods). *Biometrics & Biostatistics International Journal*, *5*(6), 210-213.
- Facione, P. A., Facione, N. C., & Gittens, C. A. (2020). What the data tell us about human reasoning. In *Critical Thinking and Reasoning* (pp. 272-297). Brill.
- Firdaus, M., & Darari, M. B. (2024). Effectiveness of case-based interactive videos in online discussion forums to improve critical thinking skills of prospective mathematics teachers. *Cognizance Journal of Multidisciplinary Studies, 4*(1), 26-38.
- Jamil, M., Aslam, M., & Ali, S. (2024). Single National Curriculum (SNC) for Social Studies (2020): Document analysis for development of critical thinking skills at the primary level. *Pakistan Journal of Law, Analysis and Wisdom*, *3*(2), 67-74.

- Jamil, M., Bookhari, T. B., & Rafiq, M. (2024). Critical thinking skills development for 21st century: An analysis of Biology curriculum. *Voyage Journal of Educational Studies (VJES)*, *4*(1), 127–138.
- Jamil, M., Bookhari, T, B., Zia, Q. (2024). Qualitative Content Analysis for Critical Thinking and Skill Development: A Case of Chemistry Curriculum. *Journal of Asian Development Studies*, *13*(1), 147-155.
- Jamil, M., Hafeez, F. A., & Muhammad, N. (2024). Critical thinking development for 21st century: Analysis of Physics curriculum. *Journal of Social & Organizational Matters*, *3*(1), 1–10.
- Jamil, M., Muhammad, Y., & Sajid, S. M. H., Zahida. (2021). Critical thinking: A qualitative content analysis of education policy and secondary school science curriculum documents. *Journal of Research and Reflections in Education*, *14*(2), 249-258.
- Kiru, E. W., Doabler, C. T., Sorrells, A. M., & Cooc, N. A. (2018). A synthesis of technology-mediated mathematics interventions for students with or at risk for mathematics learning disabilities. *Journal of Special Education Technology*, *33*(2), 111-123.
- Kwangmuang, P., Jarutkamolpong, S., Sangboonraung, W., & Daungtod, S. (2021). The development of learning innovation to enhance higher order thinking skills for students in Thailand junior high schools. *Heliyon*, *7*(6) e07309.
- Kyngäs, H. (2020). Qualitative research and content analysis. In *The application of content analysis in nursing science research* (pp. 3-11). https://doi.org/10.1007/978-3-030-30199-6_1.
- Makonye, L. (2019). *Exploring the teaching of critical thinking skills to learners to promote mathematical literacy: a case of five schools in the Zaka District of Zimbabwe*
- Mao, W., Cui, Y., Chiu, M. M., & Lei, H. (2022). Effects of game-based learning on students' critical thinking: A meta-analysis. *Journal of Educational Computing Research*, *59*(8), 1682-1708.
- Mayring, P. (2014). Qualitative content analysis: Theoretical foundation, basic procedures and software solution. In *Approaches to qualitative research in mathematics education* (pp. 365–380). Springer. http://nbn-resolving.de/urn:nbn:de:0168-ssoar-395173
- Naseer, H., Muhammad, Y., & Jamil, M. (2022). Critical Thinking Skills in Pakistan Studies textbook: Qualitative Content Analysis. *Pakistan Journal of Social Research*, *4*(3), 744-755.
- Paul, R., & Elder, L. (2019). *The thinker's guide to Socratic questioning*. Rowman & Littlefield.
- Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives*, *11*(3), 184-190.
- Safitri, M., & Aziz, M. R. (2022). *Technological Pedagogical Content Knowledge (TPCK) on mathematics learning: A literature study.* Conference Proceedings International Conference on Education Innovation and Social Science,
- Schoenfeld, A. H. (2022). Why are learning and teaching mathematics so difficult? In *Handbook of cognitive mathematics* (pp. 1-35). Springer.
- Tall, D. (2013). *How humans learn to think mathematically: Exploring the three worlds of mathematics*. Cambridge University Press.
- Tirol, S. L. (2022). Spiral Progression Approach in the Kto 12 Science Curriculum: A Literature Review. *International Journal of Education (IJE)*, *10,* 29-44. https://doi.org/10.5121/ije.2022.10403.