Mean Estimators for Sensitive Variables Using Scrambled Response Models

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Abstract

The most popular methods for sensitive study are regression estimation methods that use standard regression coefficients. When it comes to survey researchers, mean estimation is an important issue. The vast majority of population mean estimation methods that can be found in sampling theory are meant only to be utilized with non-sensitive data-sets. When the variable that is important is sensitive, such as drug usage, illegal income, abortion, exam cheating, the amount of income tax due, employee rule-breaking, etc., these estimation methods cannot operate efficiently. In this article, we propose a novel method for computing the population mean using exponential technique of robust-type regression estimators for scrambled response model (SRM) under simple random sampling (SRS). The mean square error (MSE) equation is generated using a first-order approximation and examined with existing estimating techniques in order to evaluate the efficacy of the new approach. Additionally, the proposed estimator's percentage relative efficiency (PRE) is determined compared to other estimators. The effectiveness of the proposed method is demonstrated using real data sets. According to the results, the suggested estimator performs better than other estimators in the literature.

Keywords: Robust Regression; Simple Random Sampling; Scrambled Response Model.

Introduction

In classical statistics, data are known and composed of discrete numerical values. Many writers have developed a number of estimation methods for calculating the finite population mean under classical statistics when supplementary data is present.

> According to the research, when there is a strong correlation between the Y and X, the sampling error for ratio is significantly lower than when the Y is utilized single. As a result, either less sampling is required for the ratio estimation technique, or the sample size is decreases while maintaining the same level of precision, as suggested by (Cochran, 1940).

In survey sampling, there are several ways we can improve our estimate by using supplementary data. It should be noted that the ratio, regression, and product type estimation techniques are useful when the supplementary data are available (Bulut & Zaman, 2020). However, situations can also occur where many authors develop various estimators using auxiliary data, improving the

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performance of the estimation methods. In this situation, numerous writers, including (Zaman & Kadilar, 2020), constructed a number of improved and modified estimators using auxiliary data. Auxiliary information was incorporated in a thoughtful sampling design by References (Kiregyera, 1984; Vishwakarma & Gangele, 2014; Shahbaz & Kadılar, 2016; Raza et al., 2019) in order to obtain effectiveness estimations.

Let Y be a random information that is impossible to directly access. Assume that non-sensitive supplementary information X_1 and X_2, have a positively correlation with Y. Let a scrambling data S, be independent of Y, X_1 and X_2. The question ask the respondents to repost a scrambled response for Y, given as $Z = Y + S$, and to offer a correct response for X_1 and X_2. The respondent in this model is asked to add up a random number (Y) drawn from a known distribution and his sensitive trait (S). Z stands for the observed reaction, which is $Z = Y + S$. The distribution of S and Y is represented using the same notation as before. Because S and Y are independent, the mean and variance of Z are $\mu_Z = \mu_S + \mu_Y$, $\sigma_Z^2 = \sigma_S^2 + \sigma_Y^2$. For more details see (Pollock & Bek, 1976).

Pollock and Bek (1976) suggested a three randomized response techniques for quantitative variables. Diana and Perri (2010, 2011) suggested a SRM for evaluating the average of a sensitive quantitative variable. When the study information is not sensitive, a several of researchers, including (Shahzad, 2016; Shahzad et al., 2017; Koyuncu, 2012) have developed a family of estimation methods using additional information under a SRS. Similarly, Shahzad et al. (2019) and several others have examined ratio, exponential and traditional regression estimation methods for mean estimation when research variable was sensitive. Zaman ad Bulut (2019) suggested a robust-type ratio estimation methods for non-sensitive research information. Gupta et al. (2020) suggested a novel generalized class of the estimation techniques of the variance using a linear scrambling theory. Ali et al. (2021) proposed a robust-type regression estimation methods for enhancing average estimation of sensitive information by utilizing supplementary information. Shahzad et al. (2022) developed a quantile robust-type regression estimation methods for nonsensitive and sensitive information. A novel scrambled randomized response (SRR) model under SRS has been suggested by (Narjis & Shabbir, 2023) for estimating the population average of a sensitive data in the presence of scrambled responses. Saleem et al. (2023) suggested a novel SRM for the effective evaluation of the population variance of sensitive data. Alomair and Shahzad (2023) proposed a new family of Hartley– Ross-type estimation methods for estimating the population mean using neutrosophic robust regression. Novel exponential function of scrambling response in quantitative randomized response technique was developed by (Azeem, 2023; Zaman et al., 2024) proposed an effective Hartley–Ross estimation methods of non-sensitive and sensitive data utilizing robust-type regression techniques in sample surveys. Azeem et al. (2024) developed a novel randomized model for a new estimation method for the population variance of sensitive data. Yadav and Prasad (2023) suggested a sampling theory, the exponential estimation technique is used under robust-type quantile regression techniques. Taking inspiration from Yadav and Prasad (2023) study, we have expanded their estimation technique defining a more general family of exponential robust-type quantile estimation methods for the sensitive setup. After that, we have also developed a novel class of exponential robust-type quantile estimators for sensitive setup under SRS scheme.

The remaining sections of the article are organized as follows: provide exponential robust- type quantile regression techniques in section 2. In section 3, the developed estimators are provided. In section 4, a numerical analysis is carried out. The article concludes in section 5.

Adapted Estimators under Simple Random Sampling Scheme for Scrambled Response

In this portion, following the class of exponential ratio-type regression estimation methods in SRM is shown. According to Yadav and Prasad (2023) analysis, the developed estimation method outperform then ordinary least squares (OLS) estimators in terms of effectiveness.

$$\bar{z}_{sv} = [\bar{z} + \hat{\beta}_{ols}(\bar{X} - \bar{x})]exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{\mathscr{k}}}\right] \quad (1)$$

$$\bar{z}_{vs} = [\bar{z} + \hat{\beta}_{ols}(\bar{X} - \bar{x})]exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{\mathscr{k}}}\right] \quad (2)$$

$$\bar{z}_{py} = [\bar{z} + \hat{\beta}_{ols}(\bar{X} - \bar{x})]exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{\mathscr{k}}}\right] \quad (3)$$

Where the Bowley's coefficient of skewness is expressed as $\mathcal{S}_{\mathscr{k}} = \frac{\psi_3+\psi_1-2\psi_2}{\psi_3-\psi_1}$, ψ_i is the i^{th} quartile and $\hat{\beta}_{ols} = \frac{S_{xz}}{s_x^2}$ is got by the OLS technique, where the sample variance is s_x^2, and S_{xz} is the sample covariance. To compute the MSE's of the estimation methods z_{sv}, z_{vs}, and z_{py} up-to the first order of substantial sample estimations are obtained under the following terms: $\bar{z} = \bar{Z}(1 + \xi_o)$ and $\bar{x} = \bar{X}(1 + \xi_1)$ that $E(\xi_j) = 0$ $|\xi_j| < 1\forall j = 0,1$, take the following forms:

$$z\bar{}\_sv = [Z\bar{}(1 + \xi\_o) + \beta\hat{}\_ols\ (X\bar{} - X\bar{}(1 + \xi\_1))]\ \exp[(\psi\_1\ (X\bar{} - X\bar{}(1 + \xi\_1)))/(\psi\_1\ (X\bar{} + X\bar{}(1 + \xi\_1)) + 2\mathcal{S}\_\mathscr{k})] \quad (4)$$

$$\bar{z}_{sv} = [\bar{Z}(1 + \xi_o) - \bar{X}\xi_1\hat{\beta}_{ols}]\left[1 - \frac{1}{2}\vartheta\xi_1 + \frac{3}{8}\vartheta^2\xi_1^2\right] \quad (5)$$

$$\bar{z}_{sv} = [\bar{Z} + \bar{Z}\xi_o - \bar{X}\xi_1\hat{\beta}_{ols}]\left[1 - \frac{1}{2}\vartheta\xi_1 + \frac{3}{8}\vartheta^2\xi_1^2\right] \quad (6)$$

$$\bar{z}_{sv} - \bar{Z} = \bar{Z}\left[\xi_o - \frac{1}{2}\vartheta\xi_1 - \frac{\bar{X}}{\bar{Z}}\hat{\beta}_{ols}\xi_1\right] \quad (7)$$

where $\vartheta = \frac{\psi_1\mathcal{S}_{\mathscr{k}}}{\psi_1\bar{X}+\mathcal{S}_{\mathscr{k}}}$ and we obtained the MSE Eqs. (8), (9), and (10) by taking the square of the previously given Eq. (7) and then taking expectation, and we were able to derive them. So, the MSE's expressions are given below:

$$MSE(\bar{z}_{sv}) = \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2S_x^2 + \hat{\beta}_{ols}^2S_x^2 - \gamma_1S_{xz} + \gamma_1\hat{\beta}_{ols}S_x^2 - 2\hat{\beta}_{ols}S_{xz}\right] \quad (8)$$

Similarly,

$$MSE(\bar{z}_{vs}) = \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_2^2S_x^2 + \hat{\beta}_{ols}^2S_x^2 - \gamma_2S_{xz} + \gamma_2\hat{\beta}_{ols}S_x^2 - 2\hat{\beta}_{ols}S_{xz}\right] \quad (9)$$

$$MSE(\bar{z}_{py}) = \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_3^2S_x^2 + \hat{\beta}_{ols}^2S_x^2 - \gamma_3S_{xz} + \gamma_3\hat{\beta}_{ols}S_x^2 - 2\hat{\beta}_{ols}S_{xz}\right] \quad (10)$$

Where $\gamma_1 = \frac{\psi_1\bar{Z}}{\psi_1\bar{X}+\mathcal{S}_{\mathscr{k}}}$, $\gamma_2 = \frac{\psi_2\bar{Z}}{\psi_2\bar{X}+\mathcal{S}_{\mathscr{k}}}$, $\gamma_3 = \frac{\psi_3\bar{Z}}{\psi_3\bar{X}+\mathcal{S}_{\mathscr{k}}}$.

Proposed Robust Quantile Regression Method

When using robust estimating techniques where information are impacted by outliers, robust-type of quantile regression coefficients are recommended as an alternative to OLS regression estimation methods. Three examples of estimation methods that are more resilient to outliers are provided below. Robust-type quantile regression techniques give better reliable estimates in the presence of outliers, and while applying the robust-type quantile regression technique to estimate any population average, it is not essential to eliminate outliers from the data-sets. It is significant to note that $q^{15^{th}} = 0.15, q^{25^{th}} = 0.25, q^{35^{th}} = 0.35, q^{45^{th}} = 0.45, q^{55^{th}} = 0.55, q^{65^{th}} = 0.65$, and $q^{75^{th}} = 0.75$ quantiles are used. Using scrambled response models: the (Pollock & Bek, 1976), $Z = Y + S$. By extending the idea of adapting estimator and taking inspiration from Reference [25] we propose the exponential robust quantile regression mean estimator in SRS as For Case-I, estimators are as follows in Eqs. (11), (12), (13), (14), (15), (16), and (17)

$$\bar{z}_{sv_{q(0.15)}} = \left[\bar{z} + \hat{\beta}_{(0.15)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (11)$$

$$\bar{z}_{sv_{q(0.25)}} = \left[\bar{z} + \hat{\beta}_{(0.25)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (12)$$

$$\bar{z}_{sv_{q(0.35)}} = \left[\bar{z} + \hat{\beta}_{(0.35)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (13)$$

$$\bar{z}_{sv_{q(0.45)}} = \left[\bar{z} + \hat{\beta}_{(0.45)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (14)$$

$$\bar{z}_{sv_{q(0.55)}} = \left[\bar{z} + \hat{\beta}_{(0.55)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (15)$$

$$\bar{z}_{sv_{q(0.65)}} = \left[\bar{z} + \hat{\beta}_{(0.65)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (16)$$

$$\bar{z}_{sv_{q(0.75)}} = \left[\bar{z} + \hat{\beta}_{(0.75)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (17)$$

For Case-II, estimators are as follows in Eqs. (18), (19), (20), (21), (22), (23), and (24)

$$\bar{z}_{vs_{q(0.15)}} = \left[\bar{z} + \hat{\beta}_{(0.15)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (18)$$

$$\bar{z}_{vs_{q(0.25)}} = \left[\bar{z} + \hat{\beta}_{(0.25)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (19)$$

$$\bar{z}_{vs_{q(0.35)}} = \left[\bar{z} + \hat{\beta}_{(0.35)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (20)$$

$$\bar{z}_{vs_{q(0.45)}} = \left[\bar{z} + \hat{\beta}_{(0.45)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (21)$$

$$\bar{z}_{vs_{q(0.55)}} = \left[\bar{z} + \hat{\beta}_{(0.55)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (22)$$

$$\bar{z}_{vs_{q(0.65)}} = \left[\bar{z} + \hat{\beta}_{(0.65)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (23)$$

$$\bar{z}_{py_{q(0.15)}} = \left[\bar{z} + \hat{\beta}_{(0.15)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (25)$$

$$\bar{z}_{py_{q(0.25)}} = \left[\bar{z} + \hat{\beta}_{(0.25)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (26)$$

$$\bar{z}_{py_{q(0.35)}} = \left[\bar{z} + \hat{\beta}_{(0.35)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (27)$$

$$\bar{z}_{py_{q(0.45)}} = \left[\bar{z} + \hat{\beta}_{(0.45)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (28)$$

$$\bar{z}_{py_{q(0.55)}} = \left[\bar{z} + \hat{\beta}_{(0.55)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (29)$$

$$\bar{z}_{py_{q(0.65)}} = \left[\bar{z} + \hat{\beta}_{(0.65)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (30)$$

$$\bar{z}_{py_{q(0.75)}} = \left[\bar{z} + \hat{\beta}_{(0.75)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (31)$$

$$\bar{z}_{vs_{q(0.75)}} = \left[\bar{z} + \hat{\beta}_{(0.75)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X} - \bar{x})}{\psi_2(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (24)$$

For Case-III, estimators are as follows in Eqs. (25), (26), (27), (28), (29), (30), and (31)

$$\bar{z}_{py_{q(0.15)}} = \left[\bar{z} + \hat{\beta}_{(0.15)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (25)$$

$$\bar{z}_{py_{q(0.25)}} = \left[\bar{z} + \hat{\beta}_{(0.25)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (26)$$

$$\bar{z}_{py_{q(0.35)}} = \left[\bar{z} + \hat{\beta}_{(0.35)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (27)$$

$$\bar{z}_{py_{q(0.45)}} = \left[\bar{z} + \hat{\beta}_{(0.45)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (28)$$

$$\bar{z}_{py_{q(0.55)}} = \left[\bar{z} + \hat{\beta}_{(0.55)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (29)$$

$$\bar{z}_{py_{q(0.65)}} = \left[\bar{z} + \hat{\beta}_{(0.65)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (30)$$

$$\bar{z}_{py_{q(0.75)}} = \left[\bar{z} + \hat{\beta}_{(0.75)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X} - \bar{x})}{\psi_3(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (31)$$

In Eqs. (32), (33), and (34) we generalize the developed estimation methods for each of the following three scenarios as follows:

$$\bar{z}_{sv_{q(i)}} = \left[\bar{z} + \hat{\beta}_{(i)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_1(\bar{X} - \bar{x})}{\psi_1(\bar{X} - \bar{x}) + 2\mathcal{S}_{k}}\right] \quad (32)$$

$for\ i = 0.15, 0.25, \ldots, 0.75$

$$\bar{z}_{vs_{q(i)}} = \left[\bar{z} + \hat{\beta}_{(i)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_2(\bar{X}-\bar{x})}{\psi_2(\bar{X}-\bar{x})+2\mathcal{S}_{\mathcal{k}}}\right] \tag{33}$$

$for\ i = 0.15, 0.25, \ldots, 0.75$

$$\bar{z}_{py_{q(i)}} = \left[\bar{z} + \hat{\beta}_{(i)_q}(\bar{X} - \bar{x})\right] exp\left[\frac{\psi_3(\bar{X}-\bar{x})}{\psi_3(\bar{X}-\bar{x})+2\mathcal{S}_{\mathcal{k}}}\right] \tag{34}$$

$for\ i = 0.15, 0.25, \ldots, 0.75$

To compute the MSE's of the estimation methods $\bar{z}_{sv_{q(i)}}$, $\bar{z}_{vs_{q(i)}}$, and $\bar{z}_{py_{q(i)}}$ up-to the 1st order of large sample estimating are obtained under the following error terms: $\bar{z} = \bar{Z}(1 + \xi_o)$ and $\bar{x} = \bar{X}(1 + \xi_1)$ that $E(\xi_j) = 0$ $|\xi_j| < 1 \forall j = 0,1$, take the following forms:

$$\bar{z}_{sv_{q(i)}} = \left[\bar{Z}(1 + \xi_o) + \hat{\beta}_{(i)_q}\left(\bar{X} - \bar{X}(1 + \xi_1)\right)\right] exp\left[\frac{\psi_1\left(\bar{X} - \bar{X}(1 + \xi_1)\right)}{\psi_1\left(\bar{X} + \bar{X}(1 + \xi_1)\right) + 2\mathcal{S}_{\mathcal{k}}}\right] \tag{35}$$

$$\bar{z}_{sv_{q(i)}} = \left[\bar{Z}(1 + \xi_o) - \bar{X}\xi_1\hat{\beta}_{(i)_q}\right]\left[1 - \frac{1}{2}\vartheta\xi_1 + \frac{3}{8}\vartheta^2\xi_1^2\right] \tag{36}$$

$$\bar{z}_{sv_{q(i)}} = \left[\bar{Z} + \bar{Z}\xi_o - \bar{X}\xi_1\hat{\beta}_{(i)_q}\right]\left[1 - \frac{1}{2}\vartheta\xi_1 + \frac{3}{8}\vartheta^2\xi_1^2\right] \tag{37}$$

$$\bar{z}_{sv_{q(i)}} = \left[\bar{Z} + \bar{Z}\xi_o - \bar{X}\xi_1\hat{\beta}_{(i)_q}\right]\left[1 - \frac{1}{2}\vartheta\xi_1 + \frac{3}{8}\vartheta^2\xi_1^2\right] \tag{38}$$

$$\bar{z}_{sv_{q(i)}} - \bar{Z} = \bar{Z}\left[\xi_o - \frac{1}{2}\vartheta\xi_1 - \frac{\bar{X}}{\bar{Z}}\hat{\beta}_{(i)_q}\xi_1\right] \tag{39}$$

So, the MSE's of developed estimation methods take the following forms:

$$MSE\left(\bar{z}_{sv_{q(i)}}\right) = \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(i)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(i)_q}S_x^2 - 2\hat{\beta}_{(i)_q}S_{xz}\right] \tag{40}$$

In Eq. (40), replacing i with the equal value, the MSE equation of the $\bar{z}_{sv_{q(i)}}$ can be written as:

$$MSE\left(\bar{z}_{sv_{q(i)}}\right) = \begin{cases} \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.15)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.15)_q}S_x^2 - 2\hat{\beta}_{(0.15)_q}S_{xz}\right] \\ \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.25)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.25)_q}S_x^2 - 2\hat{\beta}_{(0.25)_q}S_{xz}\right] \\ \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.35)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.35)_q}S_x^2 - 2\hat{\beta}_{(0.35)_q}S_{xz}\right] \\ \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.45)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.45)_q}S_x^2 - 2\hat{\beta}_{(0.45)_q}S_{xz}\right] \\ \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.55)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.55)_q}S_x^2 - 2\hat{\beta}_{(0.55)_q}S_{xz}\right] \\ \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.65)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.65)_q}S_x^2 - 2\hat{\beta}_{(0.65)_q}S_{xz}\right] \\ \frac{1-f}{n}\left[S_z^2 + \frac{1}{4}\gamma_1^2 S_x^2 + \hat{\beta}_{(0.75)_q}^2 S_x^2 - \gamma_1 S_{xz} + \gamma_1\hat{\beta}_{(0.75)_q}S_x^2 - 2\hat{\beta}_{(0.75)_q}S_{xz}\right] \end{cases}$$

Further the MSE of the other estimation methods are as:

$$MSE\left(\bar{z}_{py_{q(i)}}\right)=\begin{cases}\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.15)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.15)_q}S_x^2-2\hat{\beta}_{(0.15)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.25)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.25)_q}S_x^2-2\hat{\beta}_{(0.25)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.35)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.35)_q}S_x^2-2\hat{\beta}_{(0.35)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.45)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.45)_q}S_x^2-2\hat{\beta}_{(0.45)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.55)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.55)_q}S_x^2-2\hat{\beta}_{(0.55)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.65)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.65)_q}S_x^2-2\hat{\beta}_{(0.65)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(0.75)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(0.75)_q}S_x^2-2\hat{\beta}_{(0.75)_q}S_{xz}\right]\end{cases}$$

$$MSE\left(\bar{z}_{vs_{q(i)}}\right)=\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(i)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(i)_q}S_x^2-2\hat{\beta}_{(i)_q}S_{xz}\right] \tag{41}$$

In Eq. (41), replacing i with the equivalent value, the MSE equation of the $\bar{z}_{vs_{q(i)}}$ can be written as:

$$MSE\left(\bar{z}_{vs_{q(i)}}\right)=\begin{cases}\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.15)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.15)_q}S_x^2-2\hat{\beta}_{(0.15)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.25)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.25)_q}S_x^2-2\hat{\beta}_{(0.25)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.35)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.35)_q}S_x^2-2\hat{\beta}_{(0.35)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.45)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.45)_q}S_x^2-2\hat{\beta}_{(0.45)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.55)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.55)_q}S_x^2-2\hat{\beta}_{(0.55)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.65)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.65)_q}S_x^2-2\hat{\beta}_{(0.65)_q}S_{xz}\right]\\\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_2^2S_x^2+\hat{\beta}_{(0.75)_q}^2S_x^2-\gamma_2S_{xz}+\gamma_2\hat{\beta}_{(0.75)_q}S_x^2-2\hat{\beta}_{(0.75)_q}S_{xz}\right]\end{cases}$$

and

$$MSE\left(\bar{z}_{py_{q(i)}}\right)=\frac{1-f}{n}\left[S_z^2+\frac{1}{4}\gamma_3^2S_x^2+\hat{\beta}_{(i)_q}^2S_x^2-\gamma_3S_{xz}+\gamma_3\hat{\beta}_{(i)_q}S_x^2-2\hat{\beta}_{(i)_q}S_{xz}\right] \tag{42}$$

In Eq. (42), replacing i with the equivalent value, the MSE equation of the $\bar{z}_{py_{q(i)}}$ can be written as:

Numerical Study

In this section, the numerical analysis is done with the help of a real-life population to investigate the proposed estimator's performance with the existing estimators in SRS design based on MSEs and PREs using two real-world datasets. Observe that the scrambling variable $S\sim N(0,\sigma)$ for the scrambled response, as per Reference [26]. Where 10% of the additional information standard deviation, or σ, is used as the standard deviation. Take note that we use the additive $(Z = Y + S)$ model by Reference [8] both employ the same scrambling technique. PRE of an estimator can be computed through the following expressions:

$$PRE(\bar{y}_Q, \bar{y}_P) = \frac{\bar{y}_Q}{\bar{y}_P} \times 100$$

Where $P = \bar{z}_{sv_{q(i)}}, \bar{z}_{vs_{q(i)}}, \ and \ \bar{z}_{py_{q(i)}}$

$for\ i = 0.15, 0.25, 0.35, 0.45, 0.55, .065, and\ 0.7$

and

$Q = \bar{z}_{sv}, \bar{z}_{vs}, \quad and\ \bar{z}_{py}$

Real Life Population 1

We use data from (Reference [27]) about the "UScereals" data-set output that is obtainable in MASS package (R-Software) (R-core Team 2021) where,

$X =$ the weight of the sodium in grams and $Y =$ the weight in grams of the calories.

For more details see Reference [25].

Table 1: Statistics regarding population 1

$N = 65$	$\gamma_1 = 0.6351693$
$n = 20$	$\gamma_2 = 0.6351695$
$\bar{X} = 237.8384$	$\gamma_3 = 0.6351696$
$\bar{Z} = 151.0678$	$\hat{\beta}_{(0.15)_q} = 0.1788828$
$C_x = 0.549237$	$\hat{\beta}_{(0.25)_q} = 0.1578953$
$S_{xz} = 4315.994$	$\hat{\beta}_{(0.35)_q} = 0.1415899$
$S_x = 130.6296$	$\hat{\beta}_{(0.45)_q} = 0.1897326$
$S_z = 62.92084$	$\hat{\beta}_{(0.55)_q} = 0.1636686$
$\hat{\beta}_{ols} = 0.2529284$	$\hat{\beta}_{(0.65)_q} = 0.2011744$
$\rho = 0.5251032$	$\hat{\beta}_{(0.75)_q} = 0.2488642$

Real Life Population 2

To demonstrate the effectiveness of the developed estimation approach in this study, we examine a dataset of factories used in (Yadav & Prasad, 2023). We assumed from (Murthy, 1967) (Page no: 288) where,

$X =$ A region set-up with capital for eighty firms.

$Y =$ A region results information for eighty firms.

Table 2: Statistics Regarding Population-2

$N = 80$	$\gamma_1 = 18.18427$
$n = 20$	$\gamma_2 = 18.18447$
$\bar{X} = 285.125$	$\gamma_3 = 18.18466$
$\bar{Z} = 5184.928$	$\hat{\beta}_{(0.15)_q} = 5.886949$
$C_x = 0.9484593$	$\hat{\beta}_{(0.25)_q} = 5.917074$
$S_{xz} = 453694.7$	$\hat{\beta}_{(0.35)_q} = 6.066542$
$S_x = 270.4294$	$\hat{\beta}_{(0.45)_q} = 6.138274$
$S_z = 1836.119$	$\hat{\beta}_{(0.55)_q} = 6.114713$
$\hat{\beta}_{ols} = 6.203771$	$\hat{\beta}_{(0.65)_q} = 5.594267$
$\rho = 0.9137113$	$\hat{\beta}_{(0.75)_q} = 5.349324$

Table 3: The MSE of proposed and existing estimators for population-1

Estimators	MSE	Estimators	MSE
$\bar{z}_{sv}$	158.8320	$\bar{z}_{vs_{q(0.35)}}$	124.3821
$\bar{z}_{vs}$	158.8320	$\bar{z}_{vs_{q(0.45)}}$	137.4811
$\bar{z}_{py}$	158.8321	$\bar{z}_{vs_{q(0.55)}}$	130.0495
$\bar{z}_{sv_{q(0.15)}}$	134.2900	$\bar{z}_{vs_{q(0.65)}}$	140.9970
$\bar{z}_{sv_{q(0.25)}}$	128.5119	$\bar{z}_{vs_{q(0.75)}}$	157.3170
$\bar{z}_{sv_{q(0.35)}}$	124.3820	$\bar{z}_{py_{q(0.15)}}$	134.2900
$\bar{z}_{sv_{q(0.45)}}$	137.4811	$\bar{z}_{py_{q(0.25)}}$	128.5119
$\bar{z}_{sv_{q(0.55)}}$	130.0495	$\bar{z}_{py_{q(0.35)}}$	124.3821
$\bar{z}_{sv_{q(0.65)}}$	140.9970	$\bar{z}_{py_{q(0.45)}}$	137.4811
$\bar{z}_{sv_{q(0.75)}}$	157.3170	$\bar{z}_{py_{q(0.55)}}$	130.0495
$\bar{z}_{vs_{q(0.15)}}$	134.2900	$\bar{z}_{py_{q(0.65)}}$	140.9970
$\bar{z}_{vs_{q(0.25)}}$	128.5119	$\bar{z}_{py_{q(0.75)}}$	157.3170

Table 4: The MSE of proposed and existing estimators for population-2

Estimators	MSE	Estimators	MSE
$\bar{z}_{sv}$	247586.9	$\bar{z}_{vs_{q(0.35)}}$	240799.9
$\bar{z}_{vs}$	247591.9	$\bar{z}_{vs_{q(0.45)}}$	244337.3
$\bar{z}_{py}$	247596.7	$\bar{z}_{vs_{q(0.55)}}$	243172.3
$\bar{z}_{sv_{q(0.15)}}$	232062.4	$\bar{z}_{vs_{q(0.65)}}$	218214.7
$\bar{z}_{sv_{q(0.25)}}$	233514.9	$\bar{z}_{vs_{q(0.75)}}$	206982.8
$\bar{z}_{sv_{q(0.35)}}$	240795.0	$\bar{z}_{py_{q(0.15)}}$	232071.8
$\bar{z}_{sv_{q(0.45)}}$	244332.3	$\bar{z}_{py_{q(0.25)}}$	233524.3
$\bar{z}_{sv_{q(0.55)}}$	243167.4	$\bar{z}_{py_{q(0.35)}}$	240804.6

$\bar{z}_{sv_{q(0.65)}}$	218210.0	$\bar{z}_{py_{q(0.45)}}$	244342.0
$\bar{z}_{sv_{q(0.75)}}$	206978.3	$\bar{z}_{py_{q(0.55)}}$	243177.0
$\bar{z}_{vs_{q(0.15)}}$	232067.3	$\bar{z}_{py_{q(0.65)}}$	218219.1
$\bar{z}_{vs_{q(0.25)}}$	233519.8	$t_{py_{q(0.75)}}$	206987.1

Table 5: PRE of proposed estimator's w.r.t existing estimators for Population 1

Estimators	$\bar{z}_{sv}$	$\bar{z}_{vs}$	$\bar{z}_{py}$
$\bar{z}_{sv_{q(0.15)}}$	118.2754	100.9631	118.2754
$\bar{z}_{sv_{q(0.25)}}$	123.5932	123.5932	123.5932
$\bar{z}_{sv_{q(0.35)}}$	127.6969	127.6969	127.6969
$\bar{z}_{sv_{q(0.45)}}$	115.5301	115.5301	115.5300
$\bar{z}_{sv_{q(0.55)}}$	122.1320	122.1320	122.1319
$\bar{z}_{sv_{q(0.65)}}$	112.6492	112.6492	112.6492
$\bar{z}_{sv_{q(0.75)}}$	100.9631	100.9630	100.963
$\bar{z}_{vs_{q(0.15)}}$	118.2754	118.2754	118.2754
$\bar{z}_{vs_{q(0.25)}}$	123.5932	123.5932	123.5932
$\bar{z}_{vs_{q(0.35)}}$	127.6969	127.6969	127.6969
$\bar{z}_{vs_{q(0.45)}}$	115.5301	115.5301	115.5301
$\bar{z}_{vs_{q(0.55)}}$	122.1320	122.1320	122.1320
$\bar{z}_{vs_{q(0.65)}}$	112.6493	112.6492	112.6492
$\bar{z}_{vs_{q(0.75)}}$	100.9631	100.9631	100.9630
$\bar{z}_{py_{q(0.15)}}$	118.2754	118.2754	118.2754
$\bar{z}_{py_{q(0.25)}}$	123.5933	123.5932	123.5932
$\bar{z}_{py_{q(0.35)}}$	127.6969	127.6969	127.6969
$\bar{z}_{py_{q(0.45)}}$	115.5301	115.5301	115.5301
$\bar{z}_{py_{q(0.55)}}$	122.1320	122.1320	122.1320
$\bar{z}_{py_{q(0.65)}}$	112.6493	112.6493	112.6492
$\bar{z}_{py_{q(0.75)}}$	100.9631	100.9631	100.9631

Table 6: PRE of proposed estimator's w.r.t existing estimators for Population 2

Estimators	$\bar{z}_{sv}$	$\bar{z}_{vs}$	$\bar{z}_{py}$
$\bar{z}_{sv_{q(0.15)}}$	106.6898	106.6876	106.6855
$\bar{z}_{sv_{q(0.25)}}$	106.0262	106.0240	106.0219
$\bar{z}_{sv_{q(0.35)}}$	102.8206	102.8185	102.8165
$\bar{z}_{sv_{q(0.45)}}$	101.3320	101.3300	101.3280
$\bar{z}_{sv_{q(0.55)}}$	101.8175	101.8154	101.8135

$\bar{z}_{sv_{q(0.65)}}$	113.4627	113.4602	113.4579
$\bar{z}_{sv_{q(0.75)}}$	119.6198	119.6171	119.6147
$\bar{z}_{vs_{q(0.15)}}$	106.6919	106.6897	106.6876
$\bar{z}_{vs_{q(0.25)}}$	106.0283	106.0261	106.0240
$\bar{z}_{vs_{q(0.35)}}$	102.8227	102.8206	102.8186
$\bar{z}_{vs_{q(0.45)}}$	101.3341	101.3320	101.3301
$\bar{z}_{vs_{q(0.55)}}$	101.8196	101.8175	101.8155
$\bar{z}_{vs_{q(0.65)}}$	113.4650	113.4625	113.4602
$\bar{z}_{vs_{q(0.75)}}$	119.6222	119.6196	119.6171
$\bar{z}_{py_{q(0.15)}}$	106.6940	106.6918	106.6897
$\bar{z}_{py_{q(0.25)}}$	106.0303	106.0281	106.0261
$\bar{z}_{py_{q(0.35)}}$	102.8247	102.8226	102.8206
$\bar{z}_{py_{q(0.45)}}$	101.3360	101.3339	101.3320
$\bar{z}_{py_{q(0.55)}}$	101.8215	101.8194	101.8175
$\bar{z}_{py_{q(0.65)}}$	113.4671	113.4647	113.4624
$\bar{z}_{py_{q(0.75)}}$	119.6245	119.6218	119.6194

Figure 1: MSE for Population-1

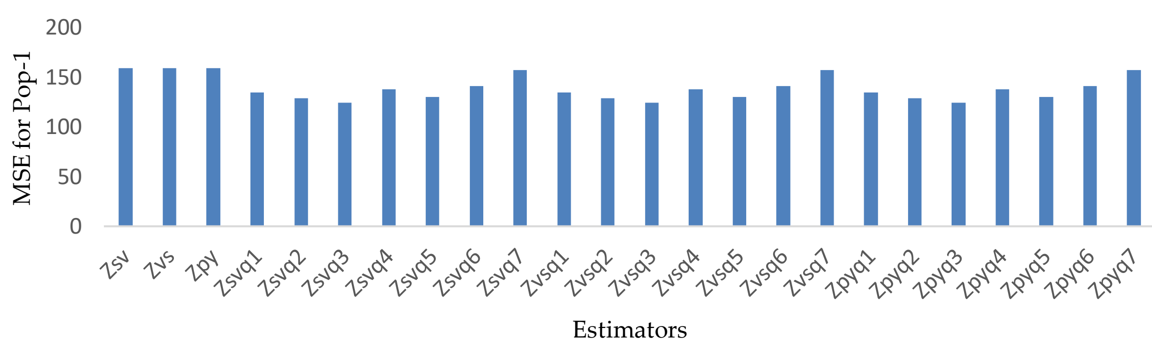


Figure 2: MSE for Population-2

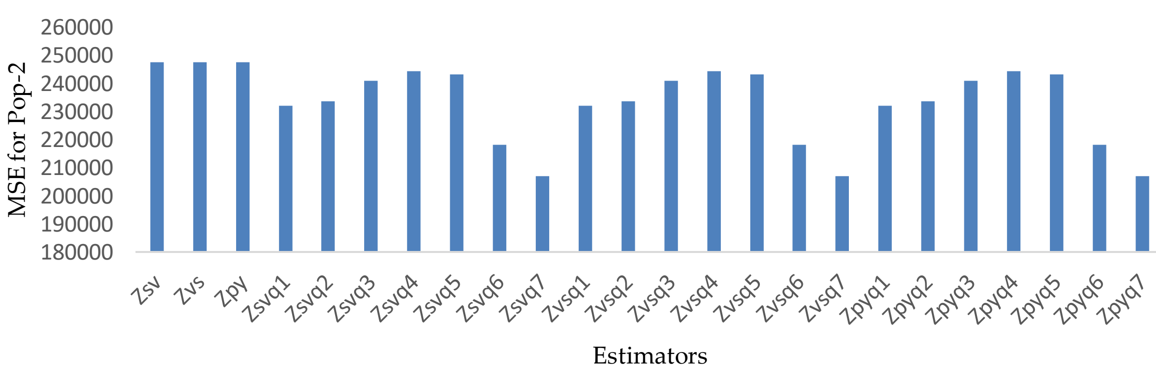


Figure 3: PRE for Population-1

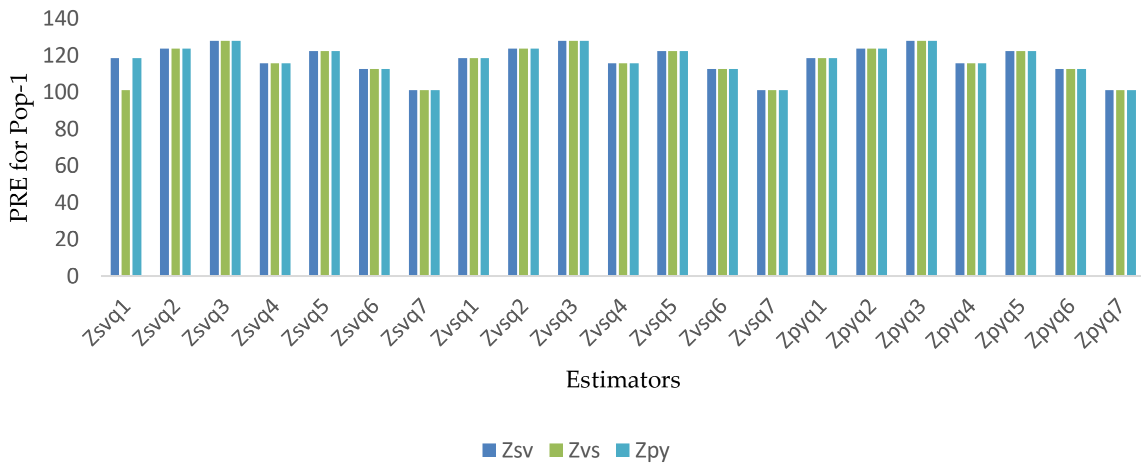
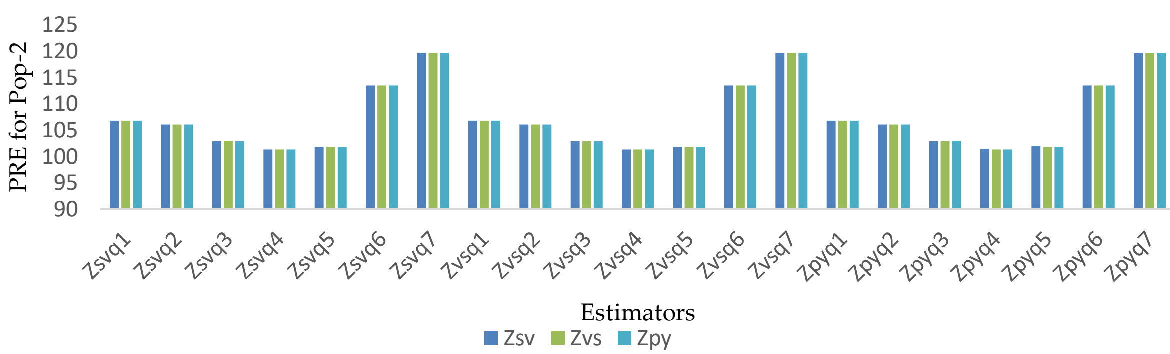


Figure 4: PRE for Population-2



Interpretation

The results of tables 3 and 4 indicate that:

For population 1 the adapted estimator $\bar{z}_{sv}$ has minimum value of MSE for $\bar{z}_{sv} = 158.8320$. The proposed estimator $\bar{z}_{sv_{q(i)}}$ for $i = 0.35$ has minimum MSE value for $\bar{z}_{sv_{q(0.35)}} = 124.3820$.

For population 2 the adapted estimator $\bar{z}_{sv}$ has minimum value of MSE for $\bar{z}_{sv} = 247586.9$. The proposed estimator $\bar{z}_{sv_{q(i)}}$ for $i = 0.75$ has minimum MSE value for $\bar{z}_{sv_{q(0.75)}} = 206978.3$.

The results of PRE of the supporting estimation methods as evaluated to other estimation methods are given in Tables 5 and 6.

The values of the PRE more than 100 shows that the developed estimators have lower MSE values than the other estimation methods.

The visual representation of MSE's and PRE's results are in provided in figures 1-4.

Conclusion

The purpose of this study is to apply the exponential robust-type quantile regression mean estimation approach under SRS. A new exponential robust regression type estimator for scrambled response has been proposed. For the scrambled regression estimation methods, we compute the PREs and MSEs. Utilizing the numerical illustration it has also been demonstrated that the proposed estimator produces smallest MSE value as compared to the adapted estimators. When data contain outliers, it is found that the proposed exponential regression type mean estimator performs better than the other estimators. This investigation is the initial step, and a whole new area is open ahead for establishing enhanced estimation methods for various types of data under different sampling strategies.

References

- Ali, N., Ahmad, I., Hanif, M., & Shahzad, U. (2021). Robust-regression-type estimators for improving mean estimation of sensitive variables by using auxiliary information. *Communications in Statistics-theory and Methods*, *50*(4), 979-992.
- Alomair, A. M., & Shahzad, U. (2023). Neutrosophic mean estimation of sensitive and non-sensitive variables with robust hartley–ross-type estimators. *Axioms*, *12*(6), 578.
- Azeem, M. (2023). Using the exponential function of scrambling variable in quantitative randomized response models. *Mathematical Methods in the Applied Sciences*, *46*(13), 13882-13893.
- Azeem, M., Salahuddin, N., Hussain, S., Ijaz, M., & Salam, A. (2024). An efficient estimator of population variance of a sensitive variable with a new randomized response technique. *Heliyon*, *10*(5).
- Bulut, H., & Zaman, T. (2022). An improved class of robust ratio estimators by using the minimum covariance determinant estimation. *Communications in Statistics-Simulation and Computation*, *51*(5), 2457-2463.
- Cochran, W. G. (1940). The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce. *The journal of agricultural science*, *30*(2), 262-275.
- Diana, G., & Perri, P. F. (2010). New scrambled response models for estimating the mean of a sensitive quantitative character. *Journal of Applied Statistics*, *37*(11), 1875-1890.
- Diana, G., & Perri, P. F. (2011). A class of estimators for quantitative sensitive data. *Statistical Papers*, *52*, 633-650.
- Gupta, S., Qureshi, M. N., & Khalil, S. (2020). Variance estimation using randomized response technique. *REVSTAT-Statistical Journal*, *18*(2), 165-176.
- Kiregyera, B. (1984). Regression-type estimators using two auxiliary variables and the variables and the model of double sampling from finite populations. *Metrika*, *31*(1), 215-226.
- Koyuncu, N. (2012). Efficient estimators of population mean using auxiliary attributes. *Applied Mathematics and Computation*, *218*(22), 10900-10905.
- Koyuncu, N., Gupta, S., & Sousa, R. (2014). Exponential-type estimators of the mean of a sensitive variable in the presence of nonsensitive auxiliary information. *Communications in statistics-Simulation and Computation*, *43*(7), 1583-1594.
- Murthy, M. N. (1967). Sampling: Theory and methods. *Calcutta, India: Statistical Publication Society*.

- Narjis, G., & Shabbir, J. (2023). An efficient new scrambled response model for estimating sensitive population mean in successive sampling. *Communications in Statistics-Simulation and Computation*, *52*(11), 5327-5344.
- Pollock, K. H., & Bek, Y. (1976). A comparison of three randomized response models for quantitative data. *Journal of the American Statistical Association*, *71*(356), 884-886.
- Raza, A., Noor-ul-Amin, M., & Hanif, M. (2019). Regression-in-ratio estimators in the presence of outliers based on redescendingm-estimator. *Journal of Reliability and Statistical Studies*, 01-10.
- Ripley, B., Venables, B., Bates, D. M., Hornik, K., Gebhardt, A., Firth, D., & Ripley, M. B. (2013). Package 'mass'. *Cran r*, *538*, 113-120.
- Saleem, I., Sanaullah, A., Al-Essa, L. A., Bashir, S., & Al Mutairi, A. (2023). Efficient estimation of population variance of a sensitive variable using a new scrambling response model. *Scientific Reports*, *13*(1), 19913.
- Shahbaz, M. Q., & Kadılar, C. (2016). Ratio estimators for population mean using robust regression in double sampling. *Gazi University Journal of Science*, *29*(4), 793-798.
- Shahzad, U., Ahmad, I., Al-Noor, N. H., Iftikhar, S., Abd Ellah, A. H., & Benedict, T. J. (2022). Särndal approach and separate type quantile robust regression type mean estimators for nonsensitive and sensitive variables in stratified random sampling. *Journal of Mathematics*, (1), 1430488.
- Shahzad, U. (2016). On the estimation of population mean under systematic sampling using auxiliary attributes. *Oriental Journal Physical Sciences*, *1*(1-2), 21-25.
- Shahzad, U., Hanif, M., Koyuncu, N., & Luengo, A. V. G. (2017). A new family of estimators for mean estimation alongside the sensitivity issue. *Journal of Reliability and Statistical Studies*, 43-63.
- Vishwakarma, G. K., & Gangele, R. K. (2014). A class of chain ratio-type exponential estimators in double sampling using two auxiliary variates. *Applied Mathematics and computation*, *227*, 171-175.
- Yadav, V. K. & Prasad, S. (2023). Exponential method of estimation in sampling theory under robust quantile regression methods. *Communications in Statistics-Theory and Methods*, 1-19.
- Zaman, T., & Bulut, H. (2019). Modified ratio estimators using robust regression methods. *Communications in Statistics-Theory and Methods*, *48*(8), 2039-2048.
- Zaman, T., & Kadilar, C. (2020). On estimating the population mean using auxiliary character in stratified random sampling. *Journal of Statistics and Management Systems*, *23*(8), 1415-1426.
- Zaman, T., Shazad, U., & Yadav, V. K. (2024). An efficient Hartley–Ross type estimators of nonsensitive and sensitive variables using robust regression methods in sample surveys. *Journal of Computational and Applied Mathematics*, *440*, 115645.