Frequency Analysis of Annual One Day Maximum Rainfall at Faisalabad, Lahore and Multan (Punjab)

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Abstract

Effective management of water systems and seepage projects depends on accurate precipitation data, particularly annual daily maximum precipitation, which is essential for designing hydraulic structures. This study conducts a frequency analysis of annual daily maximum precipitation for Faisalabad, Lahore, and Multan in Punjab, Pakistan, using over 50 years of historical data from these three stations. The analysis, performed with RAINBOW software, evaluates five probability distributions—normal, log-normal, weibull, gamma, and exponential. The fit of these distributions was assessed using Chi-square, Kolmogorov-Smirnov, and Anderson-Darling tests at significance levels of 5%, 10%, and 20%. Results indicate that the Weibull and Log-normal distributions most accurately represent the precipitation data. Based on this analysis, the study estimates the magnitude of maximum annual precipitation for return periods of 2, 5, 10, 25, 50, and 100 years. The findings aim to improve water management strategies, enhance flood and drought prevention measures, and optimize the hydraulic design of seepage structures, contributing to more effective planning and resilience in the face of extreme weather events.

Keywords: Day Annual Maximum Rainfall, Log-normal Distribution, Weibull Distribution, Gamma Distribution, Exponential Distribution.

Introduction

The measurement and intensity of rainfall are crucial for designing hydraulic structures. Global studies reveal substantial variability in total rainfall, influenced by climate conditions and the analysis period (Houghton, 1996). Given short-term precipitation fluctuations, hydraulic design and management systems focus on design precipitation events specific rainfall depths associated with certain probabilities or return periods—rather than long-term averages. Identifying the design precipitation event involves analyzing historical rainfall data, with a 30-year period generally deemed sufficient due to the variability in precipitation.

Frequency analysis is the predominant method for determining design storm events (Knowlton et al., 1984; Lane, 2002). This method estimates the likelihood of future events using techniques like the data interval analyze, ascending or descending order, and theoretical frequency distributions (Oosterbaan, 1988). Events are expressed through return periods, indicating how often a particular magnitude of annual maximum precipitation is expected to occur. Engineers select return periods based on hydrologic practices, considering factors such as potential damage, acceptable risk levels, and project lifespan. Properly estimating these values is essential to prevent damage and loss of life (Yang et al., 2010).

This study evaluates spatial and seasonal trends in temperature, precipitation, and relative humidity over 36 years (1979–2014) using Climate Forecast System Reanalysis (CFSR) datasets. The augmented Dickey–Fuller Test confirmed the stationarity of data. Trend analyses using Sen's slope, Mann–Kendall, and Cox-Stuart tests revealed significant seasonal trends, particularly in minimum temperature and precipitation during spring and autumn in Punjab, Pakistan (Syed et al., 2021).

This study forecasts temperature trends for three UK weather stations using time series analysis. Various methods, including ARIMA and ARCH/GARCH, were applied to station data. Model fitting and forecasting were conducted with R software, utilizing ACF, PACF, and ADF tests. The results showed ARIMA models effectively predicted temperatures, with comparisons highlighting the strengths of both ARIMA and ARCH/GARCH methods (Suhail et al., 2024).

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There are very few studies on this topic in Pakistan. This paper addresses this gap by analyzing the frequency of daily rainfall data at 16 locations across the country and provides insights for watershed professionals and developers on the construction of hydraulic structures in the region.

Objectives

- To determine the one-day maximum rainfall at specific stations.
- To analyze the statistical parameters of annual one-day rainfall data across three rainfall stations in Punjab, providing insights into the mean, variability, and distribution of rainfall, which are essential for effective water resource management and flood risk assessment.

Literature Review

The floods are very dangerous, and the most important property of them is the magnitude and frequency distribution of extreme rainfall to design hydraulic structures such as spillways or dams. In this firstly L-moment based regionalization in assessing the rainfall quantiles using 23 sites for Pakistan is attempted. We break the region down into three sub-regions by elevation, and test for conditional independence, stationarity, distribution, and other assumptions. The results of regional quantile estimation identified GEV, GNO and GLD distributions as the best models for predicting low- flow estimates where overall it was observed that at longer return periods (leading to greater predictive intervals) GLO model is most robust followed by those obtained using other two distribution (Shahzadi et al., 2013).

The maximum and minimum temperatures, as well as the total monthly rainfall in Punjab Pakistan were analyzed for over a period of 54 years from 1961 to 2014. By using the Mann-Kendall test and Sen's slope estimator, significant increases in temperature were found at Rawalpindi and Faisalabad while minimum temperatures increased over most stations. Commonly, rainfall trends were positive and most pronounced in summer and fall. The research implies that spring becomes warmer and summer cooler due to the gradual increase of rainfall in this area (Khattak & Ali, 2015).

The present study focused on the spatio-temporal variation of seasonal and annual rainfall across Punjab, Pakistan (1961-2015). The study detected a significant trend in dry and wet rainfall events based on the standardized precipitation index SPI at 10% significance level from nine climatic stations. A significant rise in extremely dry and wet events is observed, particularly during the last three decades, with implications for water management and agriculture (Ali et al., 2018).

For Shirazi et al. (2019) the present study, which used 33 yrs of daily temperature and rainfall data obtained from six meteorological stations in Punjab to understand recent trends with respect to extreme daily temperature and rainfall indices. Employing 14 climate indices via RClimDex software, the results show that there have been increasing trends in tropical nights (TR20) and warm nights (TN90p), while providing evidence of a decrease in cool nights with TN10p. Temperature-related indices display inconsistent trends whereas precipitation indices exhibit a rise in extreme events at some stations. The results from our analysis are not statistically significant at the 0.05% level in most cases, but they bring out such trends and their changes over time which demand better and more complete data to undertand climate change scenario in Punjab.

Climate change is a common challenge, significantly impacting many key areas of life such as changes in the distribution and magnitude of rainfall increasing floods or droughts which can threaten food security. Secondary data from the Pakistan Meteorological Department were used to assess climate change indicators in Lahore, Faisalabad and Multan located at Punjab. Important variables such as minimum and maximum temperature, annual rainfall (mm), the diurnal temperature range (DTR) The study shows substantial changes in temperature anomalies and pronounced rainfall variability observed from 1977 to 2016 which underscores the risk of this region (Khurshid & Nawaz, 2022).

Methodology

Table 1: Selected station in province Punjab, Pakistan and data availability for each station

Station ID	Station Name	Data Range
AL01	Faisalabad	1961-2020
AL02	Lahore	1961-2020
AL03	Multan	1961-2020

The design and management of drainage systems and flood control often rely on knowledge of specific rainfall events associated with particular events or return periods. This understanding is achieved through frequency analysis of historical rainfall data. The process begins with extracting annual maximum precipitation values for a given duration—in this study, one day—from long-term records for each weather station.

In this study, daily rainfall data from three stations in Punjab (Faisalabad, Lahore, and Multan) were analyzed using RAINBOW software (Raes et al., 2006). RAINBOW is a specialized tool designed for frequency analysis and data homogeneity assessment. It facilitates the extraction and analysis of annual maximum precipitation data, allowing users to estimate rainfall depths for various return periods, which is essential for designing effective irrigation and flood control systems.

RAINBOW offers several features to aid in this process. Users can select different probability distributions to model the data and evaluate their suitability through graphical methods, such as probability plots and histograms. These visual tools help in comparing the observed data with the theoretical distributions. Additionally, RAINBOW includes statistical tests like Chi-square and Kolmogorov-Smirnov, which are used to assess whether the data conform to a specific probability distribution. These tests provide a quantitative measure of the goodness-of-fit, helping to ensure that the chosen distribution accurately represents the observed rainfall data.

Tables 2, 3, and 4 present the annual maximum precipitation values for Faisalabad, Lahore, and Multan, respectively. These tables are integral to the frequency analysis process, offering detailed information on the extreme rainfall events at each station. By analyzing these values, the study provides critical insights into rainfall patterns, which are essential for effective flood risk management and irrigation system design.

Table 2: Extracted 1-day annual maximum rainfall (AL01)

Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)
1961	36.3	1976	60	1991	63.4	2006	79
1962	47	1977	76.1	1992	88	2007	68
1963	40.6	1978	179.6	1993	45	2008	78
1964	61	1979	89.6	1994	51	2009	67
1965	58.9	1980	66	1995	38	2010	91.7
1966	31.2	1981	180.3	1996	30.4	2011	91.7
1967	71.1	1982	25	1997	136	2012	48.8
1968	48.3	1983	50	1998	82	2013	130
1969	66.8	1984	62.5	1999	35	2014	77
1970	24.6	1985	37.6	2000	33	2015	45.4
1971	51.3	1986	43	2001	70	2016	62
1972	27.9	1987	63.5	2002	58.2	2017	46
1973	75.7	1988	59.7	2003	73	2018	56
1974	43.7	1989	65	2004	38	2019	41
1975	63	1990	52.8	2005	74	2020	48.4

Table 3: Extracted 1-day annual maximum rainfall (AL02)

Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)
1961	66.8	1976	211.1	1991	75.7	2006	114.6
1962	87.4	1977	85.9	1992	69.6	2007	49.4
1963	65.8	1978	95	1993	55.1	2008	80.7
1964	199.9	1979	69.3	1994	49.4	2009	49
1965	43.7	1980	207.6	1995	76.8	2010	122
1966	52.1	1981	92.7	1996	189.7	2011	122
1967	42.7	1982	67.5	1997	151.1	2012	110
1968	72.9	1983	93.8	1998	59	2013	113.1
1969	123.7	1984	60.6	1999	88.2	2014	177
1970	49.8	1985	117.4	2000	110	2015	67
1971	37.6	1986	65.3	2001	87	2016	29
1972	89.7	1987	59.1	2002	29.4	2017	54.4
1973	104.9	1988	76.9	2003	84.2	2018	139
1974	43.7	1989	123.1	2004	58	2019	74.8
1975	69.5	1990	83.1	2005	136.8	2020	38

Table 4: Extracted 1-day annual maximum rainfall (AL03)

Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)	Year	Max annually Rainfall (mm)
1961	15	1976	134.2	1991	29	2006	50.1
1962	21.6	1977	18.5	1992	127	2007	47.1
1963	26.2	1978	114.3	1993	57.8	2008	33.2
1964	25.4	1979	28	1994	72.4	2009	25
1965	294.6	1980	99.1	1995	47.5	2010	120
1966	32.5	1981	64	1996	36.2	2011	120
1967	45.7	1982	38.1	1997	31	2012	76.5
1968	20.3	1983	55.2	1998	24	2013	48.8
1969	127	1984	40.6	1999	60	2014	62
1970	50.8	1985	71.7	2000	23	2015	52.2
1971	53.3	1986	39.4	2001	83.4	2016	96.6
1972	32	1987	34.3	2002	40	2017	34
1973	31.5	1988	54.6	2003	49.5	2018	25
1974	68.6	1989	28.1	2004	48	2019	35.7
1975	38.1	1990	100	2005	70.5	2020	21.3

In probability analysis, various distributions are utilized for accurate results (Chin, 2013). This study employs four widely recognized probability distributions for frequency analysis: the normal distribution (Haan, 2002), log-normal distribution (Forbes et al., 2011), square root normal distribution, and cube root normal distribution. These distributions are chosen for their effectiveness in modeling different types of data.

One of the most common applications of probability theory in water engineering is the determination of the probability that a structure will exceed the following values, P_e, of a design event. The return period, T refers to the mean observations of years between exceedances. The Weibull method (Weibull 1939), which is theoretically more robust, is used to estimate both the probability of exceedance and the return period (Chin 2013).

$$P_e = \frac{m}{r+1} \quad (1)$$

Where r is the rank number and n is the number of observations.

The return period T in years is associated with the highest annual probability (Chin 2013).:

$$T = \frac{1}{P_e} \quad (2)$$

The four classification hypotheses used in this study were tested using two tests: the ch-squre (χ2), and the Kolmogorov–Smirnov (K-S) tests. The two goodness of fit were accomplished at three separate significance levels (α= 5%, 10% and 20%).

In general, the chi-square test compares the fit of a theoretical distribution to a given distribution (PDF). The chi-square test statistic has the following form (Montgomery & Runger, 2010):

$$chi\ square = \sum \frac{(observed-expected)^2}{expected} \quad (3)$$

A large, measured value means that there is a significant difference between the observed value and the expected value, indicating that the model fits the data poorly. Conversely, a small mean is a good fit and leads to the acceptance of a negative hypothesis (Ho), while a negative hypothesis is rejected. The Kolmogorov-Smirnov (K-S) test determines whether the sample is from a restricted normal distribution (PDF). This test is based on the maximum difference between the theoretical and empirical cumulative distribution functions (CDFs). The Kolmogorov-Smirnov test statistic is defined as (Chakravarti et al., 1967):

$$D = max\left[F(X_i) - \frac{i-1}{N}, \frac{i}{N} - F(X_i)\right] \quad (4)$$

where F is the theoretical cumulative distribution of measured values, which should be a continuous distribution, and Xi is the random sample, i= 1,2,….,n.

Results and Discussions

Basic Statistics

Table 5 presents the statistical parameters for annual one-day rainfall data from three stations in Punjab. The table shows the mean and standard deviation of rainfall at each station:

Faisalabad: The mean annual one-day rainfall is 63.4 mm, with a standard deviation of 30.7 mm. This indicates that, on average, Faisalabad experiences a one-day rainfall of 63.4 mm, with rainfall amounts varying by 30.7 mm from the average.

Lahore: The mean rainfall is 89.6 mm, and the standard deviation is 44.6 mm. Lahore experiences higher average one-day rainfall compared to Faisalabad, with greater variability in rainfall amounts.
Multan: The mean annual one-day rainfall is 57.5 mm, with a standard deviation of 43.5 mm. Multan's average rainfall is the lowest among the three stations, but its variability is comparable to that of Lahore.
In summary, Lahore has the highest average one-day rainfall and also the greatest variability, while Faisalabad has the lowest variability. Multan shows a lower average rainfall with substantial variability similar to Lahore. These variations highlight the differences in rainfall characteristics across these locations, which is important for local water resource and flood risk management.

Table 5: Statistical parameters of annual 1 day rainfall data in 3 rainfall stations in Punjab

Station Name	Mean (mm)	Standard Deviation (mm)
Faisalabad	63.4	30.7
Lahore	89.6	44.6
Multan	57.5	43.5

Statistics Test on Goodness of Fit
Tables 6, 7, and 8 present the results of the goodness-of-fit tests for various probability distributions applied to annual one-day rainfall data from Faisalabad, Lahore, and Multan stations, respectively.
Faisalabad: For Faisalabad, the Log-normal distribution performed well across all three tests. The Chi-square test ($\chi^2 = 3.8372$) and Kolmogorov-Smirnov (K-S) test ($D = 0.0769$) both indicated that the Log-normal distribution is acceptable at a significance level (SL) of 0.2. Similarly, the Anderson-Darling test ($A^2 = 0.4123$) also accepted the Log-normal distribution at an SL of 0.2. In contrast, the Normal and Exponential distributions were generally rejected by most tests.
Lahore: For Lahore, the Log-normal distribution again showed favorable results. The Chi-square test ($\chi^2 = 2.5944$), K-S test ($D = 0.0506$), and Anderson-Darling test ($A^2 = 0.1884$) all supported the Log-normal distribution at an SL of 0.2. The Normal and Exponential distributions were not supported by the tests, especially the Exponential distribution, which was consistently rejected.
Multan: For Multan, the Log-normal distribution was also acceptable across all tests. The Chi-square test ($\chi^2 = 0.6520$), K-S test ($D = 0.0705$), and Anderson-Darling test ($A^2 = 0.4218$) all indicated acceptance at an SL of 0.2. However, similar to the other locations, the Exponential distribution was rejected by all tests.
In summary, the Log-normal distribution generally provides the best fit for the rainfall data at all three stations, while the Exponential distribution consistently fails the goodness-of-fit tests.

Table 6: Results of goodness of fit for discussed distributions of Faisalabad station in Punjab

Distribution	Chi-Square	Chi-Square test Results	Kolmogorov-Smirnov	(K-S) test Results	Anderson-Darling	Anderson-Darling test Results
Normal Distribution	10.8190	Distribution is rejected at SL of 0.05	0.1571	Distribution can be accepted at SL of 0.05	2.7346	Distribution is rejected at SL of 0.05
Log Normal Distribution	3.8372	Distribution can be accepted at SL of 0.2	0.0769	Distribution can be accepted at SL of 0.2	0.4123	Distribution can be accepted at SL of 0.2
Weibull Distribution	2.4646	Distribution can be accepted at SL of 0.2	0.0781	Distribution can be accepted at SL of 0.2	1.7546	Distribution is rejected at SL of 0.2
Gamma Distribution	5.4375	Distribution can be accepted at SL of 0.2	0.1149	Distribution can be accepted at SL of 0.2	1.2625	Distribution can be accepted at SL of 0.2
Exponential Distribution	57.045	Distribution is rejected at SL of 0.01	0.3309	Distribution is rejected at SL of 0.01	10.201	Distribution is rejected at SL of 0.01

Table 7: Results of goodness of fit for discussed distributions of Lahore station in Punjab

Distribution	Chi-Square	Chi-Square test Results	Kolmogorov-Smirnov	(K-S) test Results	Anderson-Darling	Anderson-Darling test Results
Normal Distribution	6.7167	Distribution can be accepted at SL of 0.2	0.1421	Distribution is rejected with SL of 0.2	1.8907	Distribution is rejected with SL of 0.2
Log Normal Distribution	2.5944	Distribution can be accepted at SL of 0.2	0.0506	Distribution can be accepted at SL of 0.2	0.1884	Distribution can be accepted with SL of 0.2
Weibull Distribution	2.8634	Distribution can be accepted with SL of 0.2	0.0927	Distribution can be accepted with SL of 0.2	1.1377	Distribution can be accepted with SL of 0.2
Gamma Distribution	0.9209	Distribution can be accepted with SL of 0.2	0.0796	Distribution can be accepted with SL of 0.2	0.4950	Distribution can be accepted with SL of 0.2
Exponential Distribution	40.381	Distribution is rejected at SL of 0.01	0.3156	Distribution is rejected at SL of 0.01	8.4763	Distribution is rejected at SL of 0.01

Table 8: Results of goodness of fit for discussed distributions of Multan station in Punjab

Distribution	Chi-Square	Chi-Square test Results	Kolmogorov-Smirnov	(K-S) test Results	Anderson-Darling	Anderson-Darling test Results
Normal Distribution	8.9968	Distribution is rejected with SL of 0.1	0.19220	Distribution can be accepted at SL of 0.02	3.8639	Distribution can be accepted at SL of 0.01
Log Normal Distribution	0.6520	Distribution can be accepted with SL of 0.2	0.0705	Distribution can be accepted at SL of 0.2	0.4218	Distribution can be accepted at SL of 0.2
Weibull Distribution	5.3173	Distribution can be accepted with SL of 0.2	0.1071	Distribution is rejected at SL of 0.1	2.1801	Distribution can be accepted at SL of 0.2
Gamma Distribution	6.6597	Distribution can be accepted at SL of 0.2	0.1533	Distribution is rejected at SL of 0.2	1.9557	Distribution is rejected at SL of 0.1
Exponential Distribution	22.456	Distribution is rejected at SL of 0.01	0.2641	Distribution is rejected at SL of 0.01	5.2965	Distribution is rejected at SL of 0.01

Probability of Exceedance and Return Period

Table 9 presents the extreme annual 1-day rainfall depths for various return periods (2, 5, 10, 25, 50, and 100 years) based on frequency analysis conducted at three stations in Punjab, Pakistan. The analysis shows that, in Lahore, the maximum expected rainfall in one day is 89.6 mm for a 2-year return period. For a 100-year return period, the maximum one-day rainfall in Punjab is projected to be 193.5 mm, also in Lahore.

Table 9: Estimated annual 1-Day maximum rainfall corresponding to different return periods in Punjab

Stations Name	Maximum 1-Day Rainfall (mm)					
	2-Years	5-Years	10-Years	25-Years	50-Years	100-Years
Faisalabad	63.4	89.2	102.8	117.2	126.5	134.9
Lahore	89.6	127.2	146.9	167.8	181.3	193.5
Multan	57.5	94.1	113.3	133.7	146.9	158.8

Conclusion

Frequency analysis of extreme rainfall events is critical for effective management of water resources and flood risks, particularly at the basin scale. This study aims to refine our understanding of rainfall extremes by analyzing annual maximum monthly rainfall data from three stations in Punjab: Faisalabad, Lahore, and Multan. The analysis utilized five probability distributions—Normal, Log-normal, Weibull, Gamma, and Exponential—to estimate the one-day annual maximum rainfall for various return periods.

The evaluation of these distributions involved applying Kolmogorov-Smirnov, Chi-square, and Anderson-Darling tests to assess their goodness of fit. These statistical tests help determine how well each distribution fits the observed data, which is crucial for accurate risk assessment and resource management.

The results indicate that the Log-normal distribution consistently provided the best fit for the rainfall data across all three stations. Specifically, for Faisalabad, Lahore, and Multan, the Log-normal distribution was accepted by all three goodness-of-fit tests at a significance level (SL) of 0.2 or higher, making it the most reliable model for predicting extreme rainfall events. In contrast, the Normal and Exponential distributions were generally rejected by these tests. For example, the Exponential distribution failed all tests in each station, indicating poor fit for the observed rainfall data. By using the Log-normal distribution, the study estimated one-day annual maximum rainfall for return periods ranging from 2 to 100 years. This estimation provides valuable insights into potential extreme rainfall events, which is essential for designing infrastructure and implementing flood mitigation strategies. Accurate prediction of rainfall extremes enables better planning for water resource allocation and flood risk management, ultimately contributing to enhanced resilience and preparedness in the face of extreme weather events.

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Annex 1

Table A.1: Statistical test for normal distribution for the 3 rain gage stations in Punjab

Station Name	Faisalabad	Lahore	Multan
Chi-Square	10.8190	6.7167	8.9968
Chi-Square test Results	Distribution is rejected at SL of 0.05	Distribution can be accepted at SL of 0.2	Distribution is rejected with SL of 0.1
Kolmogorov-Smirnov	0.1571	0.1421	0.19220
(K-S) test Results	Distribution can be accepted at SL of 0.05	Distribution is rejected with SL of 0.2	Distribution can be accepted at SL of 0.02
Anderson-Darling	2.7346	1.8907	3.8639
Anderson-Darling test Results	Distribution is rejected at SL of 0.05	Distribution is rejected with SL of 0.2	Distribution can be accepted at SL of 0.01

Table A.2: Statistical test for Log-normal distribution for the 3 rain gage stations in Punjab

Station Name	Faisalabad	Lahore	Multan
Chi-Square	3.8372	2.5944	0.6520
Chi-Square test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution can be accepted with SL of 0.2
Kolmogorov-Smirnov	0.0769	0.0506	0.0705
(K-S) test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution can be accepted at SL of 0.2
Anderson-Darling	0.4123	0.1884	0.4218
Anderson-Darling test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution can be accepted at SL of 0.2

Table A.3: Statistical test for Weibull distribution for the 3 rain gage stations in Punjab

Station Name	Faisalabad	Lahore	Multan
Chi-Square	2.4646	2.8634	5.3173
Chi-Square test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution can be accepted with SL of 0.2
Kolmogorov-Smirnov	0.0781	0.0927	0.1071
(K-S) test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution is rejected at SL of 0.1
Anderson-Darling	1.7546	1.1377	2.1801
Anderson-Darling test Results	Distribution is rejected at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution can be accepted at SL of 0.2

Table A.4: Statistical test for Gamma distribution for the 3 rain gage stations in Punjab

Station Name	Faisalabad	Lahore	Multan
Chi-Square	5.4375	0.9209	6.6597
Chi-Square test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution can be accepted at SL of 0.2
Kolmogorov-Smirnov	0.1149	0.0796	0.1533
(K-S) test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution is rejected at SL of 0.2
Anderson-Darling	1.2625	0.4950	1.9557
Anderson-Darling test Results	Distribution can be accepted at SL of 0.2	Distribution can be accepted with SL of 0.2	Distribution is rejected at SL of 0.1

Table A.5: Statistical test for Exponential distribution for the 3 rain gage stations in Punjab

Station Name	Faisalabad	Lahore	Multan
Chi-Square	57.045	40.381	22.456
Chi-Square test Results	Distribution is rejected at SL of 0.2	Distribution is rejected at SL of 0.2	Distribution is rejected at SL of 0.2
Kolmogorov-Smirnov	0.33097	0.3156	0.26416
(K-S) test Results	Distribution is rejected at SL of 0.2	Distribution is rejected at SL of 0.2	Distribution is rejected at SL of 0.2
Anderson-Darling	10.201	8.4763	5.2965
Anderson-Darling test Results	Distribution is rejected at SL of 0.2	Distribution is rejected at SL of 0.2	Distribution is rejected at SL of 0.2

Annex 2

Figure A.1: Probability plot (CDF) for selected stations in Punjab.

Faisalabad **Lahore** **Multan**

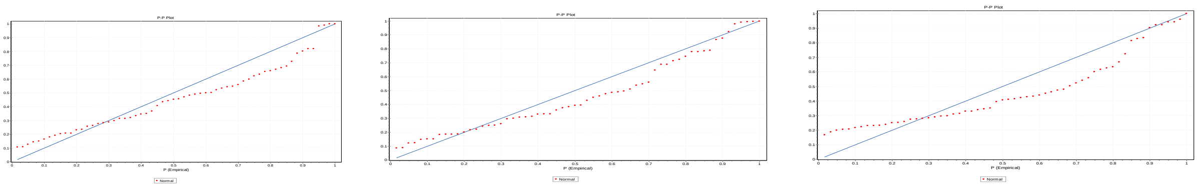

Figure A.2: Probability plot (CDF) for selected stations in Punjab.

Faisalabad **Lahore** **Multan**

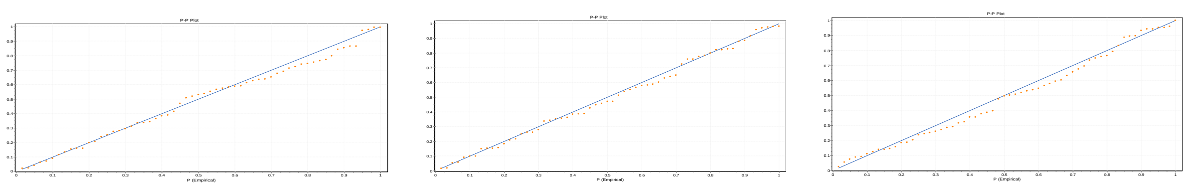

Figure A.3: Probability plot (CDF) for selected stations in Punjab.

Faisalabad **Lahore** **Multan**

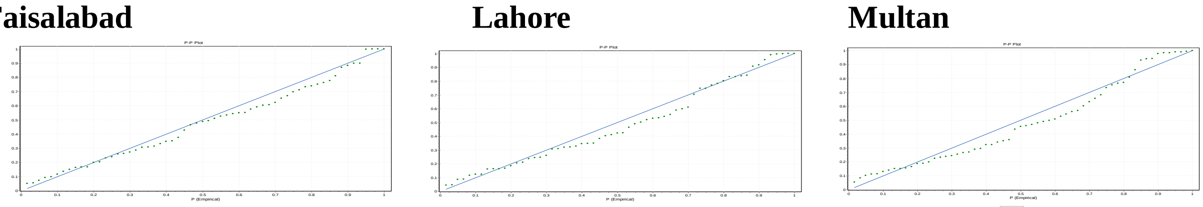

Figure A.4: Probability plot (CDF) for selected stations in Punjab.

Faisalabad **Lahore** **Multan**

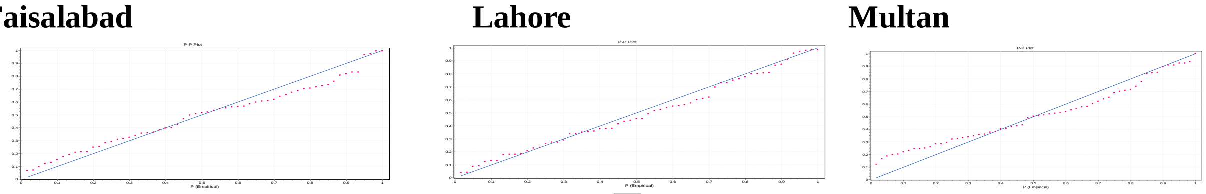

Figure A.5: Probability plot (CDF) for selected stations in Punjab.

Faisalabad **Lahore** **Multan**